

EMERGENCE OF HIGH PEAKS IN THE AXIAL VELOCITY FOR AN IDEAL MAGNETOHYDRODYNAMICS DISK CONFIGURATION

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Abstract: We study the profile of a thin disk configuration as described by an axisymmetric ideal magnetohydrodynamics steady equilibrium. We consider the disk like a differentially rotating system dominated by the Keplerian term, but allowing for a non-zero radial and vertical matter flux. As a result, the steady state allows for the existence of local peaks for the vertical velocity of the plasma particles, though it prevents the radial matter accretion rate. This ideal magnetohydrodynamics scheme is therefore unable to solve the angular momentum-transport problem, but we suggest that it provides a mechanism for the generation of matter-jet seeds.

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1. Basic Statements - In this paper, we consider an axisymmetric ideal magnetohydrodynamics (MHD) model for the disk structure surrounding a compact astrophysical object. Our approach, which differs from the standard astrophysical model for the disk morphology, is justified in view of the microphysical properties of the plasma having negligible value of the viscosity and the resistivity coefficients. Indeed, the standard model [1, 2] accounts for the angular momentum transport by postulating a non-zero viscous stress tensor appearing in the equilibrium as an effect of the internal turbulence. Despite this model is rather successful in explaining the observations, it remains affected by the non trivial shortcoming that the microscopic estimations for the viscosity coefficient give much smaller values than those ones required to fit data. This is particularly true in accreting objects like X-ray stars, where the observed rate of accretion is very high. An analogous difficulty concerns the proper balance of the force acting on electrons, when a significant infalling radial velocity occurs. The solution to this puzzle is offered by the presence of an effective resistivity of the plasma, which compensate the azimuthal Lorentz force acting on the electrons. Also such values of the resistivity coefficient are in contradiction with the microscopical estimations related to the astrophysical scales. The turbulent viscosity and resistivity coefficients can be maintained both relevant for the equilibrium by postulating a *Prandtl number* of order unity for the accreting plasma. This shortcoming of the standard model is still open to the scientific debate and leads to speak of an *anomalous* visco-resistivity of the disk plasma, rising from the so-called magnetorotational instability described by E. Velikhov [3, 4]. However, the laboratory experience on turbulent plasmas does not confirm the correlation between the internal

instabilities and the applicability of a viscoresistive MHD model with Prandtl numbers of order unity.

In our analysis, we address a different scenario as closely related to the transport profile of magnetically confined laboratory plasmas [5]. Indeed, in [6], has been shown how the ideal MHD profile of a thin disk can have a significant deviation from the Keplerian behavior for a strong vertical Lorentz force. In [7, 8] it has been demonstrated that the local configuration of a thin disk can be reduced to the profile of a ring sequence, when the electromagnetic backreaction is sufficiently intense and the vertical Lorentz force is able to confine the disk (see also [9]). Despite a solution of the angular momentum transport (*i.e.*, a non-zero radial accretion rate) is not yet available in this ideal scheme, it suggests the idea that (instead of a turbulent behavior) the disk plasma could be characterized by the formation of microstructures which might drive the typical *ballooning-mode* instabilities observed in laboratory systems. The accretion mechanism would then rely on the porosity of the plasmas near the x-points of the magnetic field configuration.

We here follow the research line traced in [6, 7, 8], but including vertical and radial matter fluxes in the ideal MHD analysis (see also [10, 11, 12]). The resulting equilibrium configuration is characterized by a vanishing net mass accretion rate, as in the previous approaches, since contributions due to the radial velocity over and below the equatorial plane cancel out. However, our local configuration permits the existence of specific disk layers where the vertical velocity can acquire a very large amplitude. This scenario suggests the possibility to realize matter jets from the disk equilibrium profile, especially in view of the implementation of this same morphology in presence of the expected plasma instabilities and this issue constitutes the main achievement of our analysis. Indeed, the link between highly magnetized disk plasmas and the jet formation (observed in the different electromagnetic bands) constitutes, in the standard model, one of the most relevant motivations for studying the angular momentum transport across these axisymmetric systems (for a proposal on the jet formation see [13]). Our result appears particularly appealing because the viscoresistive model has some difficulties to create the conditions for the jet formation, because the viscoresistive MHD scenario predicts diffusive magnetic configurations [14, 15].

2. Vertical and Radial Equilibria - To adapt the MHD equations to the axial symmetry, we introduce cylindrical coordinates $\{r, \phi, z\}$ and the fundamental variables have no dependence on ϕ . The gravitational potential of the central object writes

$$\chi(r, z) = GM_S / \sqrt{r^2 + z^2}, \quad (1)$$

where M_S is the star mass and G the Newton constant. The continuity equation, in the steady state of the disk, reads as the vanishing divergence of the matter flux $\epsilon \vec{v}$ (ϵ being the mass density and \vec{v} the velocity field) and it admits the following solution:

$$\epsilon \vec{v} = -\vec{e}_r (\partial_z \Theta)/r + \vec{e}_\phi \epsilon r \omega(r, z^2) + \vec{e}_z (\partial_r \Theta)/r, \quad (2)$$

where, $\omega(r, z^2)$ is the disk differential angular velocity (in view of the co-rotation theorem [16], it can be written as $\omega = \omega(\psi)$, $\psi(r, z^2)$ being the magnetic flux surface). Here, the generic function $\Theta(r, z)$ must be odd in z to deal with a non-zero mass accretion rate of the disk (\dot{M}_d):

$$\dot{M}_d = -2\pi r \int_{-H_0}^{+H_0} dz \epsilon v_r = 4\pi \Theta(r, H) > 0, \quad (3)$$

where H_0 is the half-depth of the disk and $v_r < 0$ the infalling radial velocity. The magnetic field reads as

$$\vec{B} = -\vec{e}_r (\partial_z \psi)/r + \vec{e}_\phi I(\psi, z)/r + \vec{e}_z (\partial_r \psi)/r, \quad (4)$$

with the function $I(\psi, z)$ allowing for the existence of poloidal currents in the disk plasma.

In order to investigate the effects induced on the disk profile by the electromagnetic reaction of the plasma, we consider a local model of the equilibrium around a radius value $r = r_0$. Thus, we split the mass density ϵ and the thermostatic pressure p into the background (barred terms) and perturbation (hatted terms) components, as $\epsilon = \bar{\epsilon}(r_0, z^2) + \hat{\epsilon}$ and, of course, $p = \bar{p}(r_0, z^2) + \hat{p}$, respectively. The same way, we express the magnetic surface function in the form $\psi = \psi_0(r_0) + \psi_1(r_0, r - r_0, z^2)$, with $\psi_1 \ll \psi_0$. In particular, the co-rotation theorem allows the representation $\omega = \omega_K + \tilde{\omega}_0 \psi_1$, where ω_K is the Keplerian term and $\tilde{\omega}_0 = \text{const}$. This form for ω holds locally, as far as $(r - r_0)$ remains sufficiently small so that the dominant deviation from the Keplerian contribution is due to ψ_1 .

When we consider the “drift ordering” for the behavior of the gradient amplitude (*i.e.*, the first-order gradients of the perturbations are of zeroth-order while the second-order ones dominate) we are able to cast the equilibrium configuration by a hierarchical ordering of the different contributions. Accordingly, the profile of the disk toroidal currents J_ϕ and the azimuthal component of the Lorentz force F_ϕ have the following form:

$$J_\phi \simeq -c (\partial_r^2 \psi_1 + \partial_z^2 \psi_1) / 4\pi r_0 , \quad (5a)$$

$$F_\phi \simeq (\partial_z I \partial_r \psi_1 - \partial_r I \partial_z \psi_1) / 4\pi r_0 . \quad (5b)$$

We now fix the equations governing the vertical and the radial equilibria of the disk by separating the fluid components from the electromagnetic backreaction (as discussed in [7, 8]). Indeed, we assume that the contribution of the toroidal magnetic field does not affect these equilibria and therefore the resulting configuration equations overlap that ones derived originally by B. Coppi in 2005. The quantity $I(\psi, z)$ plays a crucial role in the azimuthal equation only, allowing for a non-zero radial velocity even in the ideal MHD scenario.

The splitting of the MHD equations gives, for the vertical force balance, the following system of equations

$$D \equiv \bar{\epsilon}/\epsilon_0 = \exp[-z^2/H_0^2] , \quad H_0^2 \equiv 4K_B T/(m\omega_K^2) , \quad (6a)$$

$$\partial_z \hat{p} + \omega_K^2 z \hat{\epsilon} - \partial_z \psi_1 (\partial_z^2 \psi_1 + \partial_r^2 \psi_1) / 4\pi r_0^2 = 0 , \quad (6b)$$

where $\epsilon_0 = \epsilon_0(r_0) \equiv \epsilon(r_0, 0)$ and the temperature T admits the form (K_B being the Boltzmann constant):

$$2K_B T \equiv m(p/\epsilon) = m(\bar{p} + \hat{p})/(\bar{\epsilon} + \hat{\epsilon}) \equiv 2K_B(\bar{T} + \hat{T}) , \quad (7)$$

The radial equilibrium splits into the decomposition for ω (*i.e.*, $\omega \simeq \omega_K + \delta\omega \simeq \omega_0(\psi_0) + \tilde{\omega}_0 \psi_1$) and the equation

$$\begin{aligned} 2\omega_K r_0 (\bar{\epsilon} + \hat{\epsilon}) \tilde{\omega}_0 \psi_1 + \partial_r \psi_1 (\partial_z^2 \psi_1 + \partial_r^2 \psi_1) / (4\pi r_0^2) = \\ = \partial_r [\hat{p} + (\partial_r \psi_1)^2 / (8\pi r_0^2)] + \partial_r \psi_1 \partial_z^2 \psi_1 / (4\pi r_0^2) . \end{aligned} \quad (8)$$

Let us now define the dimensionless functions Y , \hat{D} and \hat{P} , in place of ψ_1 , $\hat{\epsilon}$ and \hat{p} , *i.e.*,

$$Y \equiv k_0 \psi_1 / (\partial_r \psi_0) , \quad \hat{D} \equiv \beta \hat{\epsilon} / \epsilon_0 , \quad \hat{P} \equiv \beta \hat{p} / p_0 , \quad (9)$$

where $p_0 \equiv 2K_B \bar{T}_0 \epsilon_0$ (\bar{T}_0 being $\bar{T}(r, 0)$) and

$$\beta \equiv 8\pi p_0 / B_{0z}^2 = 1/(3\epsilon_z^2) \equiv k_0^2 H_0^2 / 3 . \quad (10)$$

Here, we introduced the fundamental wave-number k_0 of the radial equilibrium, defined as $k_0 \equiv 3\omega_K^2 / v_A^2$, with $v_A^2 \equiv B_{z0}^2 / (4\pi \epsilon_0)$, recalling that $B_{z0} = (\partial_r \psi_0) / r_0$. Thus, we introduce the dimensionless radial variable defined as $x \equiv k_0(r - r_0)$, while the fundamental length in the vertical direction is taken as $\Delta \equiv \sqrt{\epsilon_z} H_0$, leading to define $u \equiv z / \Delta$. This way, the relation $D = \exp[-u^2 \epsilon_z]$ holds.

These definitions allow us to rewrite the vertical and radial equilibria as follows

$$\partial_{u^2}\hat{P} + \epsilon_z\hat{D} - 2(\partial_x^2Y + \epsilon_z\partial_u^2Y)\partial_{u^2}Y = 0, \quad (11a)$$

$$(D + \hat{D}/\beta)Y + \partial_x^2Y + \epsilon_z\partial_u^2Y + \frac{1}{2}\partial_x\hat{P} + (\partial_x^2Y + \epsilon_z\partial_u^2Y)\partial_xY = 0. \quad (11b)$$

The system above fixes two of the three unknowns Y , \hat{P} and \hat{D} and has to be completed by the azimuthal equilibrium and the electron force balance. We stress that, in setting eqs.(11), we have neglect the inertial contributions coming from the radial and the vertical components of the matter velocity, because of their small amplitude.

3. Additional Configuration Equations - The equilibrium along the toroidal symmetry of the disk writes

$$\epsilon v_r\partial_r(\omega r) + \epsilon v_z\partial_z(\omega r) + \epsilon\omega v_r = F_\phi. \quad (12)$$

Using the co-rotation theorem, *i.e.*, $\omega = \omega(\psi)$, and eqs.(4), (5b), the equation above restates as

$$\begin{aligned} \epsilon r_0^2(v_r B_z - v_z B_r) + 2\epsilon v_r \omega_K / \tilde{\omega}_0 = \\ = [\partial_z I(\partial_{r_0}\psi_0 + \partial_r\psi_1) - \partial_r I\partial_z\psi_1] / (4\pi r_0^2 \tilde{\omega}_0). \end{aligned} \quad (13)$$

In this scheme, we link the radial and vertical velocity fields to the poloidal currents. Indeed, we aim to set up a picture in which the radial matter flux is induced by the poloidal currents, *i.e.*, establishing a direct relation between v_r and F_ϕ . To this end, as well as for the consistence of the model, we have to analyze the electron force balance equation: $\vec{E} + (\vec{v}/c) \wedge \vec{B} = 0$. The balance of the Lorentz force has radial and vertical components which express the electric field in the form predicted by the co-rotation theorem [16, 7] and, since the axial symmetry requires $E_\phi \equiv 0$, the ϕ -component of the this equation retains the form

$$v_z B_r - v_r B_z = 0. \quad (14)$$

We now observe that, substituting eq.(14) in eq.(13), we get the searched relation between v_r and F_ϕ , *i.e.*,

$$2\epsilon v_r \omega_K = [\partial_z I(\partial_{r_0}\psi_0 + \partial_r\psi_1) - \partial_r I\partial_z\psi_1] / (4\pi r_0^2). \quad (15)$$

This relation, together with eq.(14), provides a system for the two unknown functions Θ and I , once ψ_1 and ϵ are given by the vertical and the radial equilibria, *i.e.*, eq.(11a) and eq.(11b), respectively.

Substituting v_r expressed by eq.(15) into the electron force balance (14), we get the following expression for v_z :

$$v_z = -\frac{(\partial_{r_0}\psi_0 + \partial_r\psi_1)F_\phi}{2\omega_K\epsilon_0\partial_z\psi_1(D + \hat{D}/\beta)}. \quad (16)$$

We stress that, addressing the splitting $\psi = \psi_0 + \psi_1$, the azimuthal Lorentz force stands as

$$F_\phi \simeq [\partial_z I(\partial_{r_0}\psi_0 + \partial_r\psi_1) - \partial_r I\partial_z\psi_1] / (4\pi r_0^2). \quad (17)$$

Expressing eq.(15) for v_r and eq.(16) for v_z via the function Θ , we get the following integro-differential compatibility condition

$$\partial_z\psi_1 [\int dz \partial_r F_\phi] = F_\phi (\partial_{r_0}\psi_0 + \partial_r\psi_1), \quad (18)$$

from which, provided ψ_1 , *i.e.*, $Y(x, u)$, from eqs.(11), one can determine the toroidal magnetic field $I(\psi, z)$.

4. Non-Linear Limit - Let us now analyze the case $Y \gg 1$, *i.e.*, when the electromagnetic reaction within the disk plasma is strong enough to significantly deform the background magnetic field. In this context, the vertical and radial equilibria are now described by eq.(11a) and

$$(D + \hat{D}/\beta)Y + \frac{1}{2}\partial_x \hat{P} + (\partial_x^2 Y + \epsilon_z \partial_u^2 Y)\partial_x Y = 0, \quad (19)$$

respectively. Clearly, we got a new configuration system for which linear terms in the perturbed magnetic surface Y have been neglected in favor of the quadratic ones. In the same limit, the azimuthal equation (15) rewrites

$$2\epsilon v_r \omega_K = F_\phi = [\partial_z I \partial_r \psi_1 - \partial_r I \partial_z \psi_1] / (4\pi r_0^2), \quad (20)$$

and eqs.(16), (18) rewritten in terms of Y as follows:

$$v_z = -[v_r \partial_x Y] / [\sqrt{\epsilon_z} \partial_u Y], \quad (21a)$$

$$\partial_u Y [\int du \partial_x F_\phi] = F_\phi \partial_x Y, \quad (21b)$$

respectively. We now look for a separable solution of the configurational system of the form

$$Y(u^2, x) = F(u^2) \sin(\alpha x), \quad (22a)$$

$$\hat{D}(u^2, x) = d(u^2) \cos(\alpha x), \quad (22b)$$

$$\hat{P}(u^2, x) = \xi(u^2) \sin^2(\alpha x) + \pi(u^2) \cos(\alpha x), \quad (22c)$$

$$F_\phi(u^2, x) = G(u^2) \sin^b(\alpha x). \quad (22d)$$

Substituting eqs.(22) in the set of equations derived above, we get ordinary differential relations in the variable u : from eqs.(11a), (19), we get the following system

$$\partial_{u^2} \pi + \epsilon_z d = 0, \quad (23a)$$

$$\pi - 2DF/\alpha = 0, \quad (23b)$$

$$\partial_{u^2} \xi + 2(-\alpha^2 F + \epsilon_z F'') \partial_{u^2} F = 0, \quad (23c)$$

$$3\epsilon_z^2 F d + \alpha \xi + \alpha F(-\alpha^2 F + \epsilon_z F'') = 0. \quad (23d)$$

Using $\epsilon = \bar{\epsilon} + \hat{\epsilon}$, eq.(20) and eq.(21a) give

$$v_r = [G \sin^b(\alpha x)] / [2\omega_K \epsilon_0 (D + d \cos(\alpha x) / \beta)], \quad (24a)$$

$$v_z = -[\alpha F \cot(\alpha x) v_r] / [F' \sqrt{\epsilon_z}], \quad (24b)$$

respectively. Finally, the compatibility equation (21b) can be recast to give the integro-differential relation:

$$bF' [\int du G] = FG \quad \Rightarrow \quad [\int du G] = g F^b, \quad (25)$$

where we have used $(...) = d(...)/du$ and $g = \text{const.}$

5. Analytical Solution - To derive an exact solution of the system fixed above, we now consider the case of large β -values for the disk plasma, *i.e.*, $\epsilon_z \ll 1$. Such analytical solution writes

$$D = 1 - \epsilon_z u^2, \quad F = A \exp[-u^2/2], \quad A \gg 1, \quad (26a)$$

$$\alpha = \sqrt{3}, \quad d = F/(\epsilon_z \sqrt{3}), \quad \xi = (3 - \epsilon_z u^2) F^2, \quad (26b)$$

$$\pi = 2(1 - \epsilon_z u^2) F / \sqrt{3}, \quad G = -bgu F^b. \quad (26c)$$

Hence the radial velocity (24a) takes the explicit form

$$v_r = -\frac{1}{2\omega_K \epsilon_0} \frac{bgA^b ue^{-bu^2/2} \sin^b(\sqrt{3}x)}{1 + \sqrt{3} \epsilon_z A e^{-u^2/2} \cos(\sqrt{3}x)}, \quad (27)$$

where, here and in the following, we neglect the term $\epsilon_z u^2$ with respect to terms proportional to A . It is remarkable that the total mass density is expressed by the relation $\epsilon = \epsilon_0(D + \hat{D}/\beta)$, *i.e.*,

$$\epsilon = \epsilon_0 [1 + \sqrt{3} \epsilon_z A e^{-u^2/2} \cos(\sqrt{3}x)], \quad (28)$$

and $\epsilon \geq 0$ implies $\sqrt{3} \epsilon_z A \leq 1$. This feature of the mass density profile (Figure 1, Plot (a)) ensures that the ring sequence, predicted in [8], is present in this regime too. Since v_r is

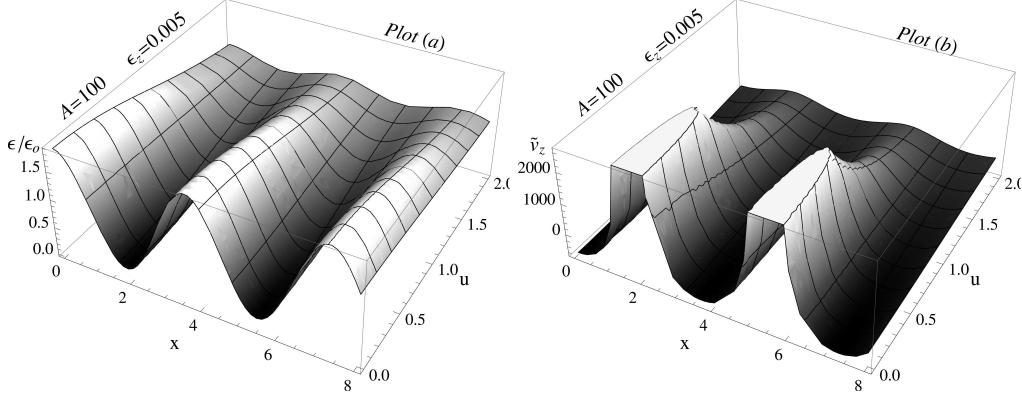


Figure 1: Behavior of ϵ/ϵ_0 (Plot (a)) and $\tilde{v}_z = v_z/v^*$ (Plot (b)) ($v^* = g/\omega_K \epsilon_0$), setting $A = 100$ and $\epsilon_z = 0.005$.

odd in u (*i.e.*, Θ is even in z) the accretion rate of the disk vanishes identically. Despite a local non-zero radial matter flux is present in the equilibrium configuration, however the compensation between the contributions over and below the equatorial plane, respectively, gives a net zero accretion of the disk.

Finally, the vertical velocity component reads as

$$v_z = -\frac{\sqrt{3/\epsilon_z}}{2\omega_K \epsilon_0} \frac{bgA^b e^{-bu^2/2} \sin^{b-1}(\sqrt{3}x) \cos(\sqrt{3}x)}{1 + \sqrt{3} \epsilon_z A e^{-u^2/2} \cos(\sqrt{3}x)}, \quad (29)$$

Let now consider the peculiar case $b = 1$ of eq.(29), near the equatorial plane. For $x\sqrt{3} \sim \pi$, we deal with a positive or negative vertical velocity (depending on the sign of Ag) having a very large amplitude, $|v_z| \gg 1$ (Figure 1, Plot (b)), as far as the condition $\epsilon_z \sqrt{3} A \sim 1$ holds (*i.e.*, in correspondence of the mass-density nodes). Such a condition is satisfied only in specific layers of the disk configuration since our model is localized at r_0 but A , g and ϵ_z vary with it.

Finally, the total term describing the magnetic surface, once we account for the first-order corrections around r_0 , reads

$$Y_t \equiv x + Y = x + A \exp[-u^2/2] \sin(\sqrt{3}x). \quad (30)$$

The ratios of the magnetic field components to the background magnetic field due to the central object, *i.e.*, $\vec{B}_0 = B_{z0} \vec{e}_z$, read as

$$\begin{aligned} B_z/B_{z0} &= \partial_x Y_t \simeq \sqrt{3} A \exp[-u^2/2] \cos(\sqrt{3}x), \\ B_r/B_{z0} &= -\sqrt{\epsilon_z} \partial_u Y_t = \sqrt{\epsilon_z} A u \exp[-u^2/2] \sin(\sqrt{3}x). \end{aligned}$$

We stress that the condition to neglect the inertial terms due to the radial and vertical velocity (despite of the presence of v_z -peaks) in the system (11) takes the following form: $Ag/(1 - \sqrt{3}\epsilon_z A) \sim \mathcal{O}(1)$. Moreover, in the regime $\epsilon_z \ll 1$ (for which our solution is derived), the condition to deal with a positive mass density, *i.e.*, $\sqrt{3}\epsilon_z A \leq 1$, ensures the positivity of the total pressure too.

6. Astrophysical Characterization - The configuration scheme we adopted above is appropriate to describe a local radial layer (a small circular crown) of a thin accretion disk since all the morphological prescriptions of this astrophysical system are included. The choice of the background magnetic surface as a function of r_0 only, models a dipole magnetic-field of the central object, as far as the disk is sufficiently thin and we take $\omega_K \propto \psi_0^3$. The gravitational field is correctly represented in the thin disk approximation, for the limit of an high Keplerian angular velocity to have a gravostatic confinement of the vertical profile of the disk, *i.e.*, $\omega_K^2 z \sim \mathcal{O}(1)$. The idea that the poloidal (radial and vertical) matter fluxes are taken small with respect to the toroidal Keplerian motion is a natural approximation, valid in the standard model too. The vertical velocity must be smaller than the azimuthal one in a thin disk, because of the background vertical equilibrium, while the radial motion is required perturbative since its presence must preserve the steady character of the matter distribution. In the standard model the amplitude of v_r is determined by the turbulent viscosity coefficient, while here it is fixed via the self-consistency of the electron force balance equation. Indeed, we deal with a vanishing net accretion, but, from a physical point of view, we could speak of a weak constant rate of matter infall, since this fact do not affect our main goal about the vertical matter jets. In this ideal plasma scenario, we can reach very high self-consistent magnetic fields and we show how they can induce intense axial matter fluxes. This picture is commonly believed the basic mechanism of the emission from high energy astrophysical sources (such as Gamma Ray Bursts or Active Galactic Nuclei), but its setting is still rather questioned. We offer here an alternative explanation, surprisingly relying only on the morphology of ideal and highly non-linear MHD structures.

7. Conclusions - We have shown how the steady state of an accretion disk, described by ideal MHD, is compatible with the presence of significant radial and vertical matter fluxes. Such matter currents are unable to account for a non-zero accretion rate, but they are relevant in view of the instabilities that can be driven by their existence. In particular, we outlined, as the main issue of our study, that there are conditions under which strong vertical matter fluxes are locally allowed, especially near the equatorial plane. This feature deserves attention because dealing with matter jets along the symmetry axis is one of the expected features of the disk configurations, but it is hard to be recovered in the standard viscoresistive model.

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